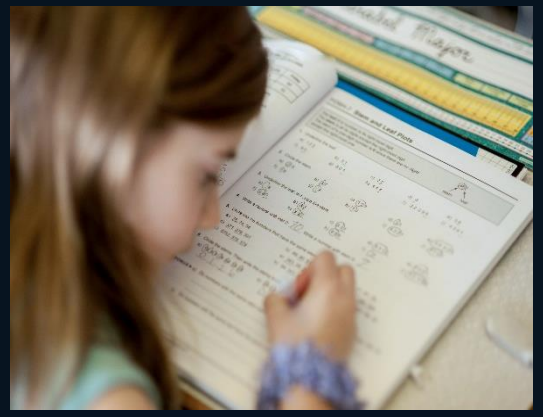


Our Mathematical Minds:

Some surprising trends in the science of learning math

Dr. John Mighton

Mathematician & JUMP Math Founder

People have more potential in mathematics than we realize.

EVIDENCE for ESSA

SCIENTIFIC AMERICAN MIND

ASPEN BRAIN FORUM
Advancing Global Breakthroughs in Brain Science

Math is **Accessible**

PNAS

Origins of the brain networks for advanced mathematics in expert mathematicians

Marie Amalric[a,b,1] **and Stanislas Dehaene**[a,c,1]

[a]Cognitive Neuroimaging Unit, Commissariat à l'Énergie Atomique et aux Énergies Alternatives, Direction des Sciences du Vivant/Institut d'Imagerie Biomédicale, INSERM, NeuroSpin Center, Université Paris-Sud and Université Paris-Saclay, 91191 Gif-sur-Yvette, France; [b]Institut de Formation Doctorale, Université Pierre-et-Marie-Curie, Université Paris 06, Sorbonne Universités, 75005 Paris, France; and [c]Collège de France, 75005 Paris, France

This contribution is part of the special series of Inaugural Articles by members of the National Academy of Sciences elected in 2010.

Contributed by Stanislas Dehaene, March 2, 2016 (sent for review January 19, 2016; reviewed by Daniel Ansari and Martin Monti)

INAUGURAL ARTICLE

SEE COMMENTARY

LEARNING THEORY

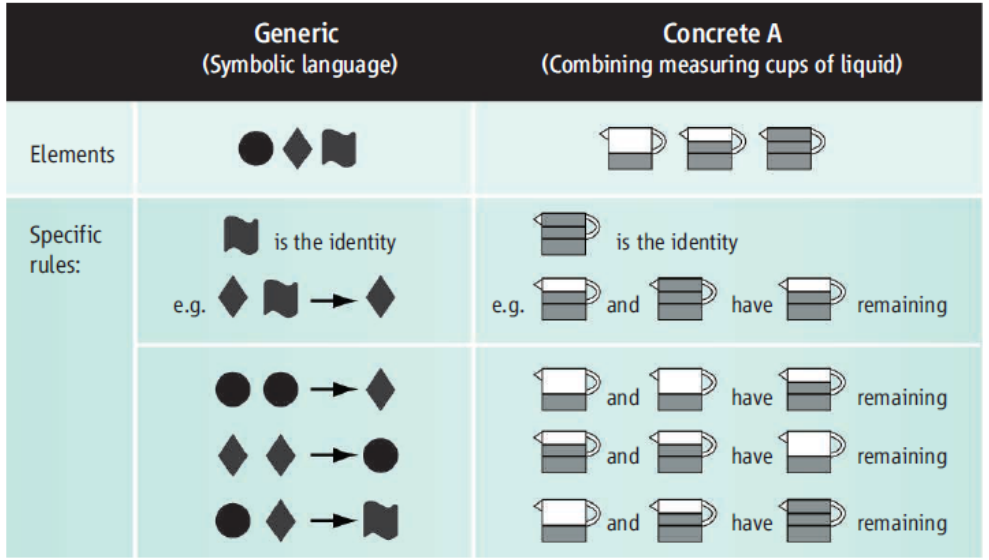
The Advantage of Abstract Examples in Learning Math

Undergraduate students may benefit more from learning mathematics through a single abstract, symbolic representation than from learning multiple concrete examples.

Jennifer A. Kaminski,* Vladimir M. Sloutsky, Andrew F. Heckler

Abstract knowledge, such as mathematical knowledge, is often difficult to acquire and even more difficult to apply to novel situations (*1*–*3*). It is widely believed that a successful approach to this challenge is to present the learner with multiple concrete and highly familiar examples of the to-be-learned concept. For instance, a mathematics instructor teaching simple probability theory may present probabilities by randomly choosing a red marble from a bag containing red and blue marbles and by rolling a six-sided die. These concrete, familiar examples instantiate the concept of probability and may facilitate learning by connecting the learner's existing knowledge with new, to-be-learned knowledge. Alternatively, the concept can be instantiated in a more abstract that communicates minimal extraneous information) may result in better knowledge transfer than learning multiple concrete, contextualized instantiations.

In experiment 1, undergraduate college students learned one or more instantiations of (*6*). The elements were three images of measuring cups containing varying levels of liquid (see figure, below). Participants were told they needed to determine a remaining amount when different measuring cups of liquid are combined. Concrete B and C instantiations were constructed similarly, with story lines and elements that would assist learning. The same mathematical rules were presented in slices of pizza or tennis balls in a container, rather than portions of a measuring cup of liquid (*9*). Eighty study participants were assigned to one of four learning conditions: Generic 1, Concrete 1, Concrete 2, or Concrete 3, with participants learning



Generic and concrete instantiations of a mathematical group.

Children's acquisition of fraction knowledge from concrete versus generic instantiations

Jennifer A. Kaminski (kaminski.16@osu.edu)
Center for Cognitive Science, Ohio State University
210A Ohio Stadium East, 1961 Tuttle Park Place, Columbus, OH 43210, USA

Vladimir M. Sloutsky (sloutsky.1@osu.edu)
Center for Cognitive Science, Ohio State University
208C Ohio Stadium East, 1961 Tuttle Park Place, Columbus, OH 43210, USA

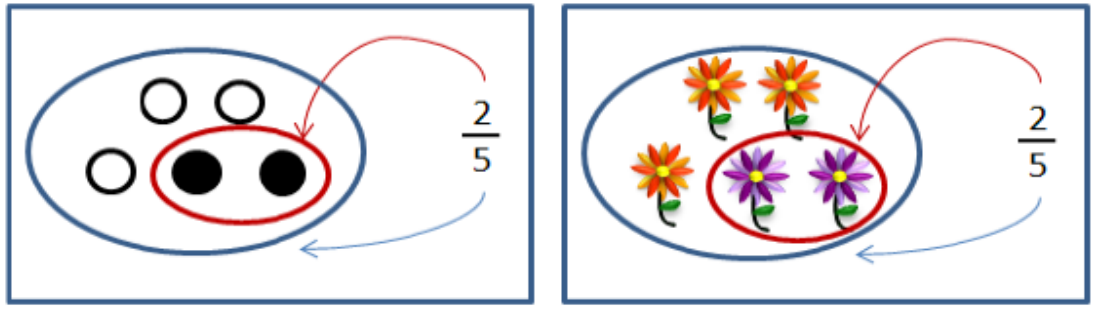


Figure 1: Example of labeling a proportion with a fraction from the training phase (Generic condition on left, Concrete condition on right).

nature communications

Article | Open access | Published: 05 November 2019

The Eighty Five Percent Rule for optimal learning

Robert C. Wilson ✉, Amitai Shenhav, Mark Straccia & Jonathan D. Cohen

Nature Communications **10**, Article number: 4646 (2019) | Cite this article

Abstract

Researchers and educators have long wrestled with the question of how best to teach their clients be they humans, non-human animals or machines. Here, we examine the role of a single variable, the difficulty of training, on the rate of learning. In many situations we find that there is a sweet spot in which training is neither too easy nor too hard, and where learning progresses most quickly. We derive conditions for this sweet spot for a broad class of learning algorithms in the context of binary classification tasks. For all of these stochastic

Academic Hierarchies & Failure

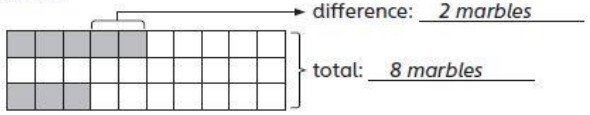
"… students as young as first grade are well aware of the different treatment that relatively low and high achieving students receive from a teacher."

- (Stipek 1997)

OA3-I2 Parts and Totals

I. Shade boxes to show the number of marbles. Then find the total and the difference.

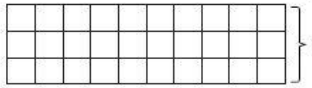
a) 5 green marbles
3 blue marbles



difference: *2 marbles*

total: *8 marbles*

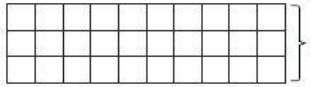
b) 4 green marbles
6 blue marbles



difference: _____

total: _____

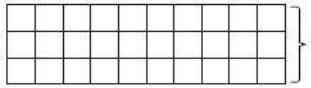
c) 8 green marbles
4 blue marbles



difference: _____

total: _____

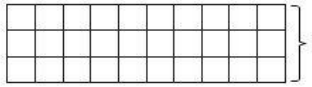
d) 9 green marbles
4 blue marbles



difference: _____

total: _____

e) 3 green marbles
8 blue marbles



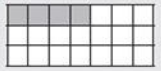
difference: _____

total: _____

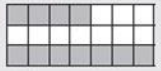
4 green marbles
3 more blue marbles than green

To draw the diagram:

Step 1: Shade the amount you know.

green 

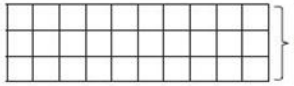
Step 2: Find the other amount.

green
blue 

2. Draw the diagram. Then fill in the blanks.

a) 5 green marbles
2 more blue marbles than green marbles

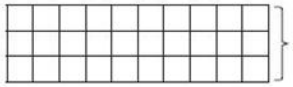
difference: _____

green
blue 

total: _____

b) 4 blue marbles
3 more green marbles than blue marbles

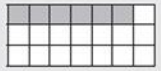
difference: _____

green
blue 

total: _____

Sometimes you know the larger amount.

6 green marbles

green 

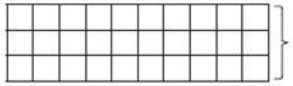
4 more green marbles than blue marbles

green
blue 

3. Draw the diagram. Then fill in the blanks.

7 green marbles
3 more green marbles than blue marbles

difference: _____

green
blue 

total: _____

Scaffolding Fluency

OA3-13 More Parts and Totals

1. Fill in the table.

	Green Marbles	Blue Marbles	Total	Difference
a)	3	5	8	*2 more blue marbles than green*
b)	2	9		
c)	4		6	
d)		2	7	
e)	6		10	
f)	3			1 more blue marble than green
g)		2		1 more green marble than blue
h)		4		1 more blue marble than green
i)	7	2		5 more green marbles than blue
j)		5		4 more green marbles than blue
k)		12		6 more blue marbles than green
l)	12	35		
m)	35			20 more green marbles than blue

3. Fill in the table. Circle the number in the table that answers the question.

		Red	Green	Total	Difference
a)	Kate has 3 green fish and 4 red fish. How many fish does she have altogether?	4	3	(7)	1
b)	Bill has 4 green fish and 6 red fish. How many fish does he have altogether?				
c)	Mary has 8 green fish and 2 more green fish than red fish. How many fish does she have?				
d)	Peter has 19 fish. He has 15 green fish. How many red fish does he have?				
e)	Hanna has 8 green fish and 3 fewer red fish than green fish. How many fish does she have?				
f)	Ken has 22 red fish and 33 green fish. How many more green fish does he have?				

4. Alice has 3 science books and 4 art books. How many books does she have?

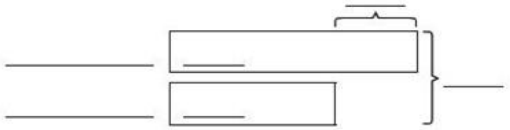
5. Marco has 5 pets. 3 are cats. The rest are dogs. How many dogs does he have?

6. Ed has 25 red apples. He has 14 more green apples than red apples. How many apples does he have?

7. There are 25 students in a class. 16 of the students are girls.
   a) How many students are boys?
   b) How many more girls are there than boys?

4. There are 24 students in a class. 14 are boys.

a) Fill in the blanks.

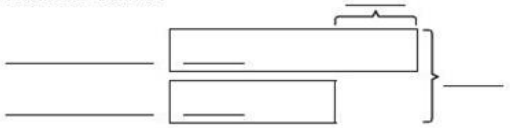


b) How many girls are in the class? _____

c) How many more boys than girls are there? _____

5. Alan has 5 more US stamps than Canadian stamps.
He has 12 Canadian stamps.

a) Fill in the blanks.



b) How many stamps does he have in total? _____

6. Sally rode her bike 252 miles to raise money for charity.
Kevin rode his bike 57 miles.

a) Draw a tape diagram to show this information.

b) How much farther did Sally ride?

c) How many miles did they ride altogether?

BONUS ▶ Hint: Use a tape diagram with 3 bars.

a) A store sold 8 books on Friday.
They sold 5 fewer books on Thursday than on Friday.
They sold 4 more books on Saturday than on Friday.
How many books did the store sell on the three days?

b) Ivan has 12 green apples.
He has 7 more red apples than green apples.
He has 3 fewer yellow apples than red apples.
How many apples does he have altogether?

The Impact