



Fractions Are Foundational

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Why Do I Need Fractions Anyway?



Fractions Are Common In Real Life Scenarios



$$r = \left(1 + \frac{i}{n}\right)^n - 1$$

Stated interest rate : 5%
Compounding frequency : monthly
Effective interest rate : ?

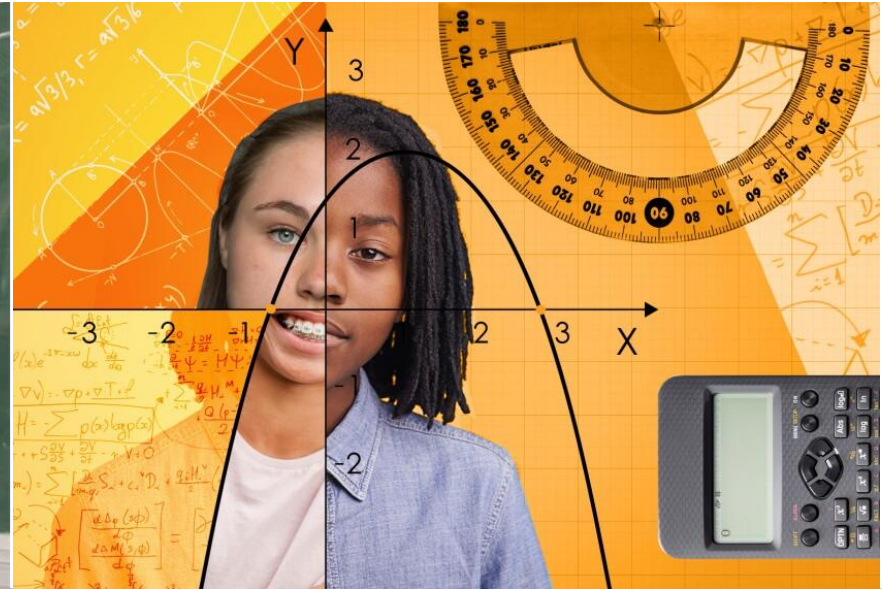
$$r = \left(1 + \frac{0.05}{12}\right)^{12} - 1 = 0.0512 = 5.12\%$$
$$r = 5.12\%$$

Key Building Block For Algebra -- Gateway To STEM Vocations



Recent Survey Of 1,000 Algebra 1 Teachers Report Fractions A Top Barrier

“When students come to middle school there is a dominant idea that a fraction is two different whole numbers written in some weird format that indicates a relationship.”



Frequent Presence Of Fractions In Algebraic Expressions


$$\frac{x+3}{6} = \frac{2}{3}$$

(Note: Red arrows indicate cross-multiplication from the numerator of the first fraction to the denominator of the second, and vice versa.)

$$(x+3)3 = (2)6$$
$$3x + 9 - 9 = 12 - 9$$
$$3x/3 = 3/3$$

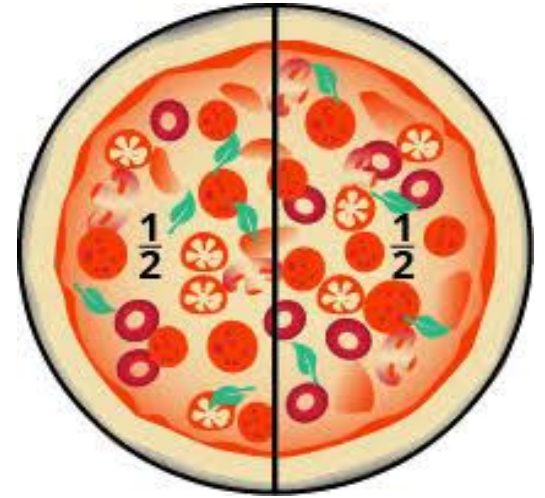
$x = 1$

wikiHow

$$\frac{2}{5} = \frac{x}{20}$$
$$x = 8$$


wikiHow to Calculate Ratios

Reactions To Fractions



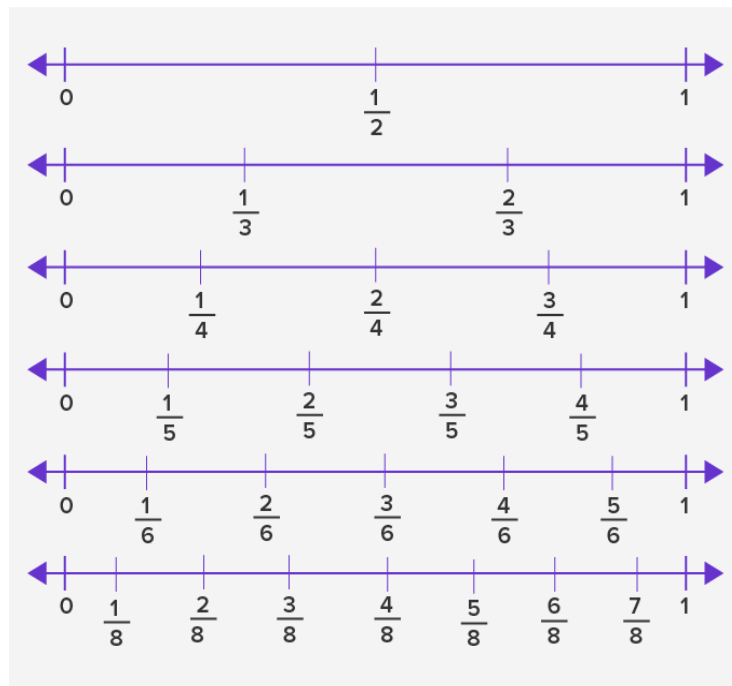
$\frac{1}{2} + \frac{2}{5} =$ i hate Fractions

Why Are Fractions Hard For Many Students?

$$\frac{3}{4} = \frac{9}{12}$$

$$\frac{1}{4} > \frac{1}{6} > \frac{1}{8}$$

$$\frac{2}{4} < \frac{6}{7}$$



Example Of Whole Number Bias!

In the 1980s, A&W tried to compete with the McDonald's Quarter Pounder by selling a $\frac{1}{3}$ pound burger at a lower cost. The product failed, because most customers thought $\frac{1}{4}$ pound was bigger.




T·H·I·R·D

Is The Word

A&W has bigger, badder third pound[®] burgers — Our Double Cheeseburger and Bacon double cheeseburger. A sizzler hot with two 100% pure beef patties freshly cooked to perfection, covered in rich, golden melted cheese. Filled high with garden-fresh lettuce, real ripe tomatoes, and pickles. Even crispy bacon if you like.


Top it off with an ice cold, frosty mug of A&W Root Beer. It's a super meal. And bring in the coupon below and save hearty appetite for bigger, badder third pound[®] burgers.

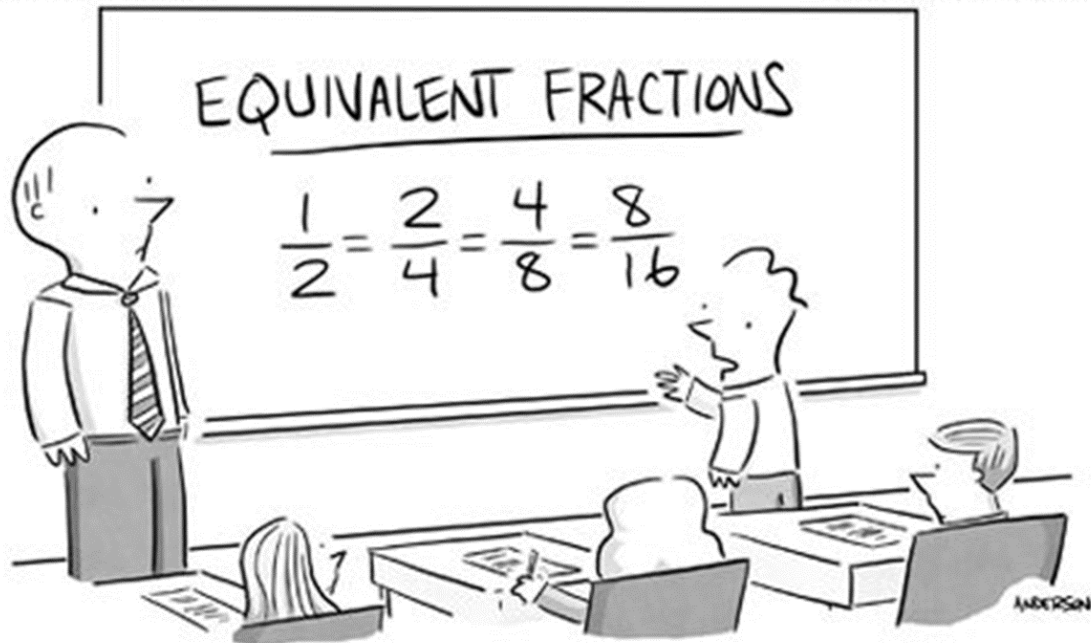
Because The Third is The Word ... at A&W.

Rediscover  We're really cookin' now.

Special Offer: \$0.00!

(Dealer Incentive)





"I understand they all have the same value, but I have to tell you, the ones on the right feel like more bang for your buck."

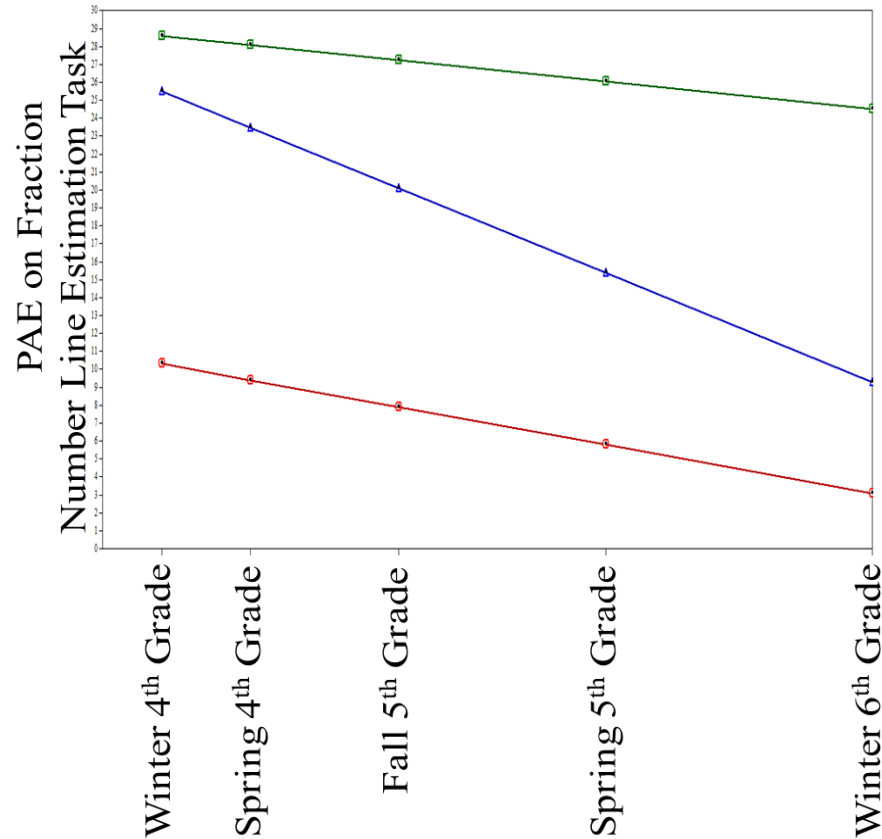
Can You Estimate the Location of this Fraction on a 0 to 2 Number Line?

$$\frac{7}{6}$$



Many Students Show Little Growth In Fractions Magnitude Understanding Between 4th & 6th Grades

(Resnick, et al., 2017)



Class 3:
Starts inaccurate; ends inaccurate (33%)

Class 2:
Starts inaccurate; ends accurate (26%)

Class 1:
Starts accurate; ends accurate (42%)

PAE = mean percent absolute error, higher score less accurate

Fractions Are HARD, Even For Teachers!

1. Stella ran $\frac{3}{4}$ mile and stopped for a drink. Then she ran another $\frac{1}{4}$ mile.
How many miles did she run altogether?

$$\frac{4}{12} \text{ mile(s)}$$

Student used the “butterfly method” (cross multiplication) to solve an addition problem

2. Elijah started the race and walked $\frac{3}{4}$ mile. He realized his hat was missing, so he walked back $\frac{1}{4}$ mile and found it. How many miles from the start is he now?

$$\frac{3}{4} - \frac{1}{4} = \frac{1}{3}$$

$$\frac{1}{3} \text{ mile(s)}$$

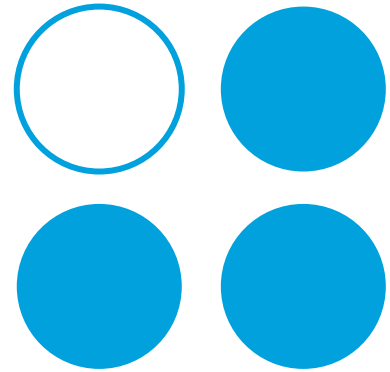
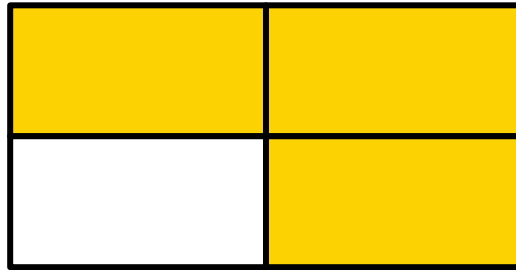
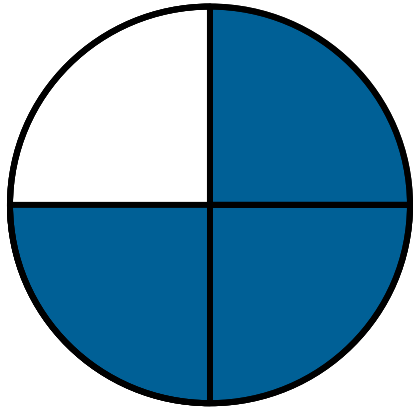
Student used the “butterfly method” (cross **subtraction**) to solve a subtraction problem

Add or Subtract with Unlike Denom.

$$\begin{array}{r} 14 \\ \hline 2 \\ 5 \end{array} + \begin{array}{r} 20 \\ \hline 4 \\ 7 \end{array} = \frac{34}{35}$$

Butterfly Method
ET/3

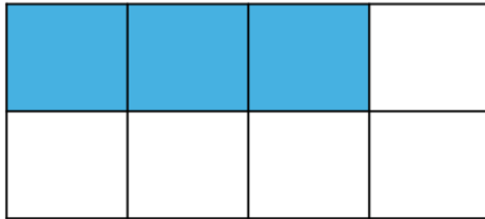
Typical Fraction Instruction For Students Emphasizes Part-Whole Understandings



(Siegler, et al., 2015; Gersten, et al., 2017)

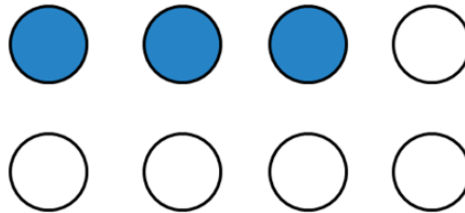
Sometimes Parts in the Whole Don't Match the Denominator

Shade $\frac{3}{4}$ of the boxes.



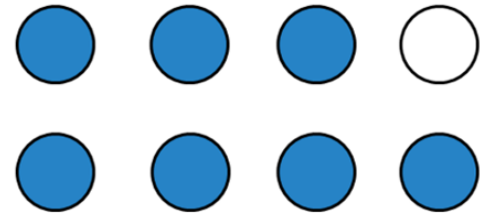
Attend to numerator only

Shade in $\frac{3}{4}$ the 8 circles.



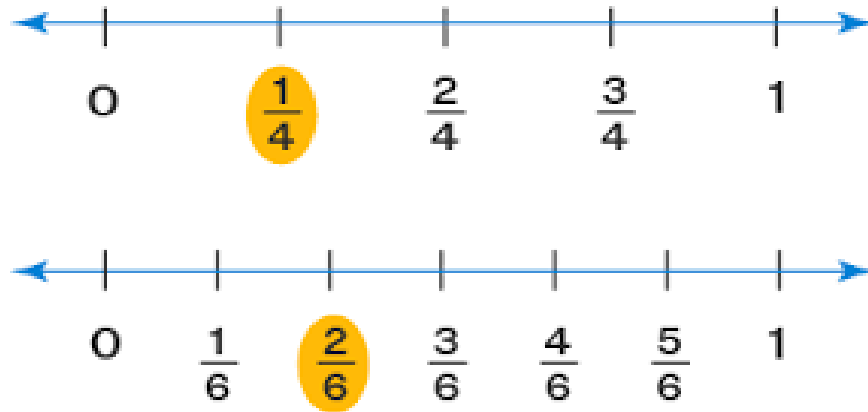
Attend to numerator only

Shade in $\frac{3}{4}$ the 8 circles.

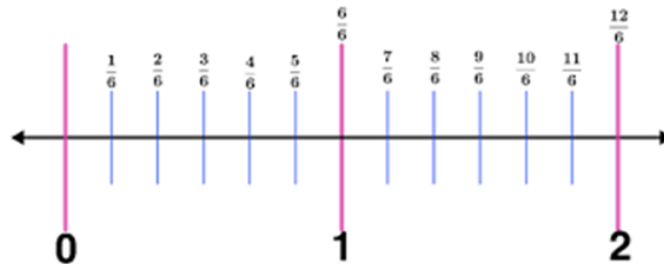


Attend to numerator and denominator as separate whole numbers

Number Line Most Helpful Model For Thinking About Fractions As Numbers



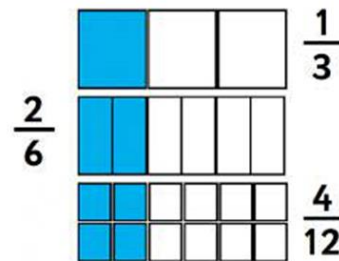
Major Components: Fraction Sense Intervention



Fractions As Numbers With Magnitudes

Fraction Equivalence And Ordering

Fraction Arithmetic & Applications



$$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$

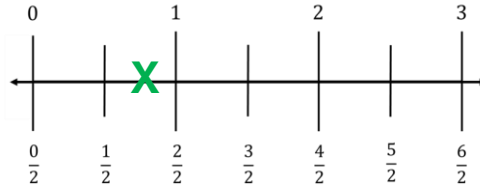
(Jordan, et al., 2024)

Understanding of $\frac{3}{4}$ With Different Strategies

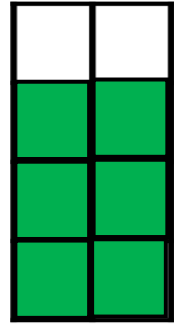
$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$



How long is the pencil? $\frac{3}{4}$ inches



Put an x where $\frac{3}{4}$ is on
the number line above.



Shade $\frac{3}{4}$ of the boxes.

Same Fractions, Different Operations

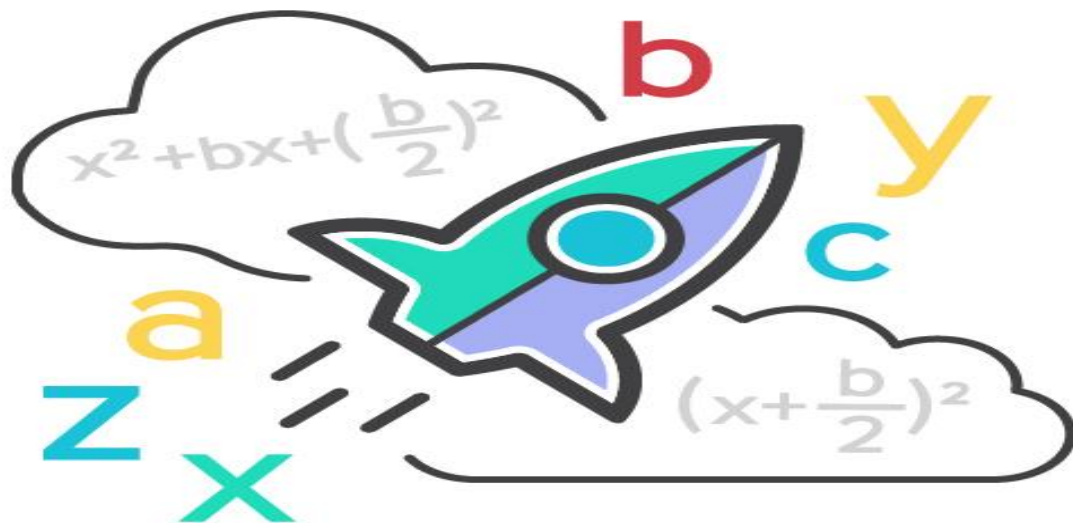
$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$$

$$\frac{1}{4} - \frac{1}{4} = 0$$

$$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$$\frac{1}{4} \div \frac{1}{4} = 1$$

Fraction Sense ➡ Algebra Sense ➡ Vocational Opportunities



Thank You!

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